Review Session 25 February 2011

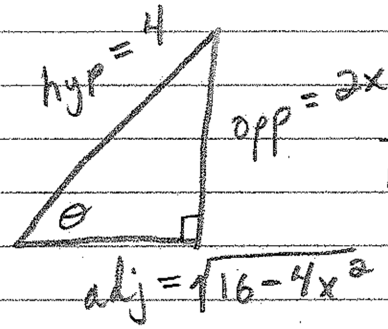
8.4.25

$$\int \sqrt{16-4x^2}\,dx$$

TRIG SUBSTITUTION

Type 1: $\sqrt{a^2-u^2}$

$$= \int \sqrt{4^2-(2x)^2}$$



$\frac{opp}{hyp} = \sin$

$$= \int (4\cos\theta)(2\cos\theta\,d\theta)$$

$opp^2 + adj^2 = hyp^2$

$\cos\theta = \frac{adj}{hyp}$

$\cos\theta = \frac{\sqrt{16-4x^2}}{4}$

$4\cos\theta = \sqrt{16-4x^2}$

$\sin\theta = \frac{2x}{4}$

$\sin\theta = \frac{x}{2}$

$x = 2\sin\theta$

$\frac{d}{d\theta}[x] = \frac{d}{d\theta}[2\sin$

$\frac{dx}{d\theta} = 2\cos\theta$

$dx = 2\cos\theta\, d$

$$= \int 8\cos^2\theta\,d\theta$$

$$= 8\int \left[\frac{1+\cos(2\theta)}{2}\right] d\theta$$

$$= 4\int (1+\cos 2\theta)\,d\theta$$

$$= 4\int d\theta + 4\int \cos(2\theta)\,d\theta$$

$$= 4\theta + 4\int \cos(u)\left(\frac{du}{2}\right)$$

$$= 4\theta + 2(\sin u) + C = 4\theta + 2\sin(2\theta) + C$$

Solve for θ:

$x = 2\sin\theta$

$\frac{x}{2} = \sin\theta$ Take Arcsin of both sides

$\arcsin\left(\frac{x}{2}\right) = \arcsin(\sin\theta)$

$\arcsin\left(\frac{x}{2}\right) = \theta$

Rewrite $\sin 2\theta$ in terms of x

$\sin 2\theta = 2\cos\theta\sin\theta$ Substitute $\cos\theta$ & $\sin\theta$

$= \cancel{2}\left(\frac{\sqrt{16-4x^2}}{4}\right)\left(\frac{x}{\cancel{2}}\right)$

$= \frac{x\sqrt{16-4x^2}}{4}$

So, $4\theta + 2\sin(2\theta) + C$

$= 4\arcsin\left(\frac{x}{2}\right) + \cancel{2}\,\frac{x\sqrt{16-4x^2}}{\cancel{4}\,2} + C$

$$\boxed{= 4\arcsin\left(\frac{x}{2}\right) + \frac{x\sqrt{16-4x^2}}{2} + C}$$

8.2 | 31) $\int t\csc(t)\cot(t)\,dt$

$$= \int t\csc(t)\cot(t)\,dt$$
$$= \int u\,dv$$
$$= (t)(-\csc(t)) - \int(-\csc t)(dt)$$
$$= -t\csc(t) + \int \csc(t)\,dt$$
$$\boxed{= -t\csc(t) - \ln|\csc(t) + \cot(t)| + C}$$

let $u = t$

$\frac{du}{dt} = 1$

$du = dt$

$\frac{dv}{dt} = \csc(t)\cot(t)$

$dv = \csc(t)\cot(t)\,dt$

$v = -\csc(t)$

8.5.25

$$\int \frac{x}{16x^4 - 1}\,dx$$

$$\frac{x}{16x^4-1} = \frac{x}{(4x^2-1)(4x^2+1)} = \frac{x}{(2x-1)(2x+1)(4x^2+1)}$$

$$\frac{x}{(2x-1)(2x+1)(4x^2+1)} = \frac{A}{(2x-1)} + \frac{B}{(2x+1)} + \frac{Cx+D}{4x^2+1} \quad [(2x-1)(2x+1)(4x^2$$

Multiply Both sides by denominator on left side.

$$x = A(2x+1)(4x^2+1) + B(2x-1)(4x^2+1) + Cx+D(2x-1)(2x$$

$$x = A(8x^3+2x+4x^2+1) + B(8x^3+2x-4x^2-1) + 4Cx^3 - Cx + D4x^2 - $$

$$x = 8Ax^3 + 2Ax + 4Ax^2 + A + 8Bx^3 + 2Bx - 4Bx^2 - B + 4Cx^3 - Cx + 4Dx^2 - $$

$$0x^3 + 0x^2 + 1x + 0 = \text{''} \quad \text{''}$$

$$0x^3 = (8A + 8B + 4C)\,x^3$$
$$0x^2 = (4A - 4B + 4D)\,x^2$$
$$1x = (2A - 2B - C)\,x$$
$$0 = (A - B - D)\cdot 1$$

MATRIX

$$\begin{array}{cccccc} A & B & C & D & & = \# \\ 8 & 8 & 4 & 0 & \vdots & 0 \\ 4 & -4 & 0 & 4 & \vdots & 0 \\ 2 & 2 & -1 & 0 & \vdots & 1 \\ 1 & -1 & 0 & -1 & \vdots & 0 \end{array}$$

Calculator

$A = 1/8$

$B = 1/8$

$C = -1/2$

$D = 0$

$$\int \frac{x}{16x^4-1}\,dx$$

$$= \int \left[\frac{\frac{1}{8}}{2x-1} + \frac{\frac{1}{8}}{2x+1} + \frac{-\frac{1}{2}x}{4x^2+1} \right] dx$$

$$= \frac{1}{8}\int \frac{1}{2x-1}\,dx + \frac{1}{8}\int \frac{1}{2x+1}\,dx - \frac{1}{2}\int \frac{x}{4x^2+1}\,dx$$

$u = 2x-1$; $\frac{du}{2} = dx$ $\quad\Big\{\quad$ $z = 2x+1$; $\frac{dz}{2} = dx$ $\quad\Big\{\quad$ $w = 4x^2+1$; $\frac{dw}{8x} = dx$

$$= \frac{1}{8}\int \frac{1}{u}\left(\frac{du}{2}\right) + \frac{1}{8}\int \frac{1}{z}\left(\frac{dz}{2}\right) - \frac{1}{2}\int \frac{\cancel{x}}{w}\left(\frac{dw}{8\cancel{x}}\right)$$

$$= \frac{1}{16}\int \frac{du}{u} + \frac{1}{16}\int \frac{dz}{z} - \frac{1}{16}\int \frac{dw}{w}$$

$$= \frac{1}{16}\ln|v| + \frac{1}{16}\ln|z| - \frac{1}{16}\ln|w| + C$$

$$= \frac{1}{16}\left[\ln|2x-1| + \ln|2x+1| - \ln|4x^2+1|\right] + C$$

$$= \frac{1}{16}\left[\ln\frac{|(2x-1)(2x+1)|}{4x^2+1}\right] + C$$

$$= \frac{1}{16}\left[\ln\frac{|4x^2-1|}{4x^2+1}\right] + C$$

8.1.37

$$\int \frac{\ln(x^2)}{x}\,dx$$

$\ln x^2 = 2\ln x$

$$= \int \frac{2\ln x}{x}$$

$$= 2\int \frac{\ln x}{x}$$

let $u = \ln x$

$du = \frac{1}{x} \cdot dx$

$x \cdot du = dx$

$$= 2\int \frac{u}{\cancel{x}}(\cancel{x}\,du)$$

$$= 2\int u\,du$$

$$= \cancel{2}\left[\frac{u^2}{\cancel{2}}\right] + C$$

$$= u^2 + C$$

$$= \ln^2 x + C$$

8.1.21

$$\int \frac{t^2 - 3}{(-t^3 + 9t + 1)} \, dt$$

let $u = -t^3 + 9t + 1$

$$\frac{du}{dt} = -3t^2 + 9$$

$$\frac{du}{dt} = -3(t^2 - 3)$$

$$\frac{du}{-3(t^2 - 3)} = dt$$

$$= \int \frac{\cancel{t^2 - 3}}{u} \left[\frac{du}{-3(\cancel{t^2 - 3})} \right]$$

$$= -\frac{1}{3} \int \frac{du}{u}$$

$$= -\frac{1}{3} \ln |u| + C$$

$$= -\frac{1}{3} \ln |-t^3 + 9t + 1| + C$$

8.4.41

$$\int \operatorname{arcsec}(2x)\,dx \quad , \quad x \geq 1/2$$

$$= \int 1 \cdot \operatorname{arcsec}(2x)\,dx$$

$$= \int u\,dv = uv - \int v\,du$$

$$= (\operatorname{arcsec}(2x))(x) - \int \cancel{x} \cdot \frac{1}{\cancel{x}\sqrt{4x^2-1}}\,dx$$

let $u = \operatorname{arcsec}(2$

$$\frac{du}{dx} = \frac{1}{|2x|\sqrt{4x^2-1}} \cdot (2$$

$$du = \frac{1}{|x|\sqrt{4x^2-1}} \cdot dx$$

$$du = \frac{1}{x\sqrt{4x^2-1}}\,dx$$

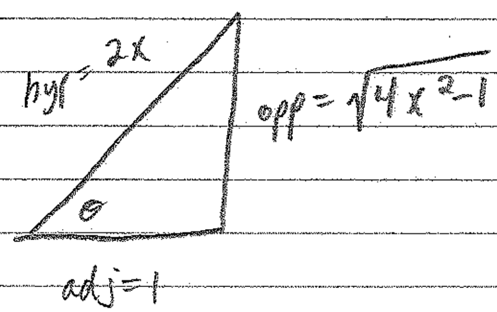
$$\frac{dv}{dx} = 1$$

$$dv = dx$$

$$v = x$$

$$= x \operatorname{arcsec}(2x) - \int \frac{1}{\sqrt{4x^2-1}}\,dx$$

TRIANGLE $\rightarrow$ Type III : $\sqrt{v^2 - a^2}$



$$\frac{2x}{1} = \frac{hyp}{adj}$$

$$2x = \sec\theta$$

$$x = \frac{1}{2}\sec\theta$$

$$\frac{d}{d\theta}\left[x\right] = \frac{d}{d\theta}\left[\frac{1}{2}\sec\theta\right]$$

$$\frac{dx}{d\theta} = \frac{1}{2}\sec\theta\tan\theta$$

$$dx = \frac{1}{2}\sec\theta\tan\theta\,d\theta$$

$$\frac{1}{\sqrt{4x^2-1}} = \frac{adj}{opp}$$

$$\frac{1}{\sqrt{4x^2-1}} = \cot(\theta)$$

$$\tan\theta = \frac{opp}{adj} = \frac{\sqrt{4x^2-1}}{1} = \sqrt{4x^2-1}$$

So,

$$\int \operatorname{arcsec}(2x)\,dx$$

$$= x\operatorname{arcsec}(2x) - \int (\cancel{\cot\theta})\left(\frac{1}{2}\sec\theta\cancel{\tan\theta}\right)d\theta$$

$$= x\operatorname{arcsec}(2x) - \frac{1}{2}\int \sec\theta\,d\theta$$

$$= x\operatorname{arcsec}(2x) - \frac{1}{2}\left[\ln\left|\sec\theta + \tan\theta\right|\right] + C$$

Convert θ back to x

$$= x\operatorname{arcsec}(2x) - \frac{1}{2}\left[\ln\left|2x + \sqrt{4x^2-1}\right|\right] + C$$

8.2.35

$$\int e^{2x}\sin x\,dx$$

$$= \int u\,dv = uv - \int v\,du$$

$$= (\sin x)\left(\tfrac{1}{2}e^{2x}\right) - \int \left(\tfrac{1}{2}e^{2x}\right)(\cos x\,dx)$$

let $u = \sin x$

$du = \cos x\,dx$

let $dv = e^{2x}\,dx$

let $z = 2x$

$dz = 2\,dx$

$\frac{dz}{2} = dx$

$dv = e^{z}\frac{dz}{2}$

$\int dv = \frac{1}{2}\int e^{z}\,dz$

$v = \frac{1}{2}e^{z} + C$

$v = \frac{1}{2}e^{2x}$

$$= \frac{1}{2}e^{2x}\sin x - \frac{1}{2}\int e^{2x}\cos x\,dx$$

let $u = \cos x$

$du = -\sin x\,dx$

$\frac{dv}{dx} = e^{2x}$

$v = \frac{1}{2}e^{2x}$

$$\int e^{2x}\sin x\,dx = \frac{1}{2}e^{2x}\sin x - \frac{1}{2}\left[(\cos x)\left(\tfrac{1}{2}e^{2x}\right) - \int\left(\tfrac{1}{2}e^{2x}\right)(-\sin x\,dx)\right]$$

$$= \frac{1}{2}e^{2x}\sin x - \frac{1}{4}e^{2x}\cos x - \frac{1}{4}\int e^{2x}\sin x\,dx$$

¡LOOP!

$$\int e^{2x}\sin x\,dx = \frac{1}{2}e^{2}\sin x - \frac{1}{4}e^{2x}\cos x - \frac{1}{4}\cancel{\int e^{2x}\sin x\,dx}$$

$$+\frac{1}{4}\int e^{2x}\sin x\,dx \qquad\qquad +\frac{1}{4}\cancel{\int e^{2x}\sin x\,dx}$$

$$\frac{\cancel{4}}{\cancel{5}}\cdot\frac{5}{\cancel{4}}\int e^{2x}\sin x\,dx = \left[\frac{1}{2}e^{2x}\sin x - \frac{1}{4}e^{2x}\cos x\right]\frac{4}{5} + C$$

MULTIPLY each side by 4/5

$$\int e^{2x}\sin x\,dx = \frac{4}{5}\left[\frac{1}{2}e^{2x}\sin x - \frac{1}{4}e^{2x}\cos x\right] + C$$

$$= \frac{2}{5}e^{2x}\sin x - \frac{1}{5}e^{2x}\cos x$$

8.2.39

$$y' = xe^{x^2}$$

$$dx \cdot \frac{dy}{dx} = xe^{x^2} \cdot dx$$

$$dy = xe^{x^2}\,dx$$

$$\int dy = \int xe^{x^2}\,dx$$

$$y = \int xe^{x^2}\,dx$$

let $z = x^2$

$dz = 2x\,dx$

$\frac{dz}{2x} = dx$

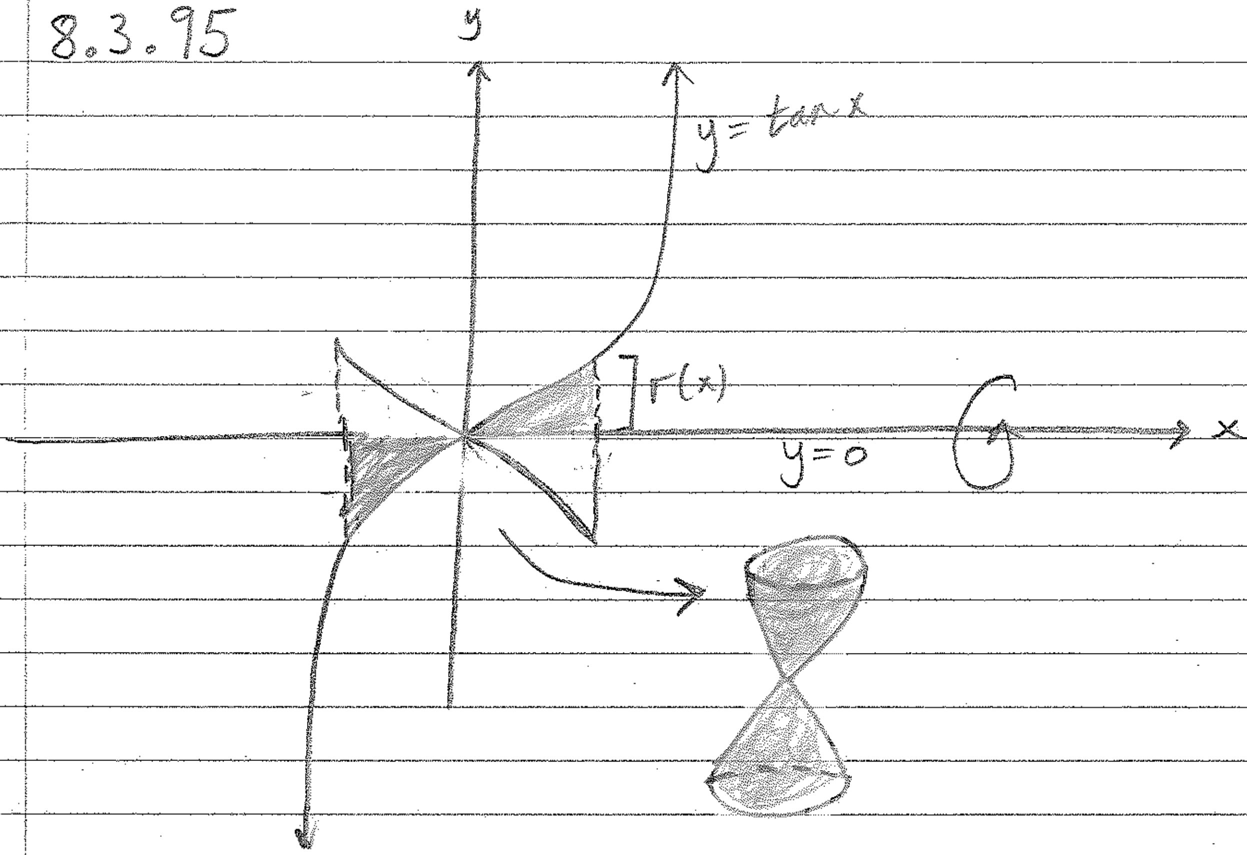
$$= \int \cancel{x} e^{z}\left(\frac{dz}{2\cancel{x}}\right)$$

$$= \frac{1}{2}\int e^{z}\,dz$$

$$= \frac{1}{2}e^{z} + C$$

$$y = \frac{1}{2}e^{x^2} + C$$

8.3.95



$$\text{VOLUME} = \pi \int_{-\pi/4}^{\pi/4} (\tan x)^2 \, dx$$

$$= 2\pi \int_0^{\pi/4} \tan^2 x \, dx$$

$$= 2\pi \int_0^{\pi/4} (\sec^2 x - 1) \, dx$$

$$= 2\pi \int_0^{\pi/4} \sec^2 x \, dx - 2\pi \int_0^{\pi/4} 1 \, dx$$

$$= 2\pi \Big[\tan x \Big]_0^{\pi/4} - 2\pi \Big[x \Big]_0^{4}$$

$$= 2\pi \Big[\tan x - x \Big]_0^{4}$$

ENDPOINTS ARE THE SAME

$$= 2\pi\left\{\left[\left(\tan \pi/4\right) - \pi/4\right] - \left[\tan 0 - 0\right]\right\}$$

$$= 2\pi\left[1 - \pi/4\right] \approx 1.348$$